Journal of Applied Mechanics and Technical Physics, Vol. 45, No. 1, pp. 31–36, 2004

EFFECT OF ALLOWANCE FOR SMALL-SCALE PERTURBATIONS OF THE CARRIER PHASE ON PROPERTIES OF THE SYSTEM OF EQUATIONS OF A TWO-FLUID FLOW WITH INCOMPRESSIBLE PHASES

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UDC 532.54:532.529.5

The effect of the fluctuating components of kinetic energy and stress tensor of the carrier phase, which were previously obtained by the cell technique, on the properties of the system of equations of a gas–liquid flow with incompressible phases is considered. It is shown that the characteristic properties of this system and also the possibility of modeling the Zuber–Findlay empirical relation are determined by the tensor of fluctuating stresses of the carrier phase.

Key words: *two-phase flow, fluctuating components, numerical solution.*

Introduction. Modeling of multiphase disperse flows is known to involve equations obtained by the method of averaging of the classical hydrodynamic equations [1] or by variational methods [2]. The final form of the system has not been yet established. As was indicated in [1], further refinement of expressions for the stress tensor, interphase forces, etc., is needed. The problem of refinement of the form of equations of heterogeneous flows is largely associated with modeling of effects caused by perturbations introduced into the carrier-phase flow by the motion of disperse particles. In the system of hydrodynamic equations, the effect of such small-scale perturbations is taken into account by fluctuating terms similar to Reynolds stresses in modeling of turbulent motion of the fluid. In calculating the fluctuating terms, the parameters characterizing the distribution of disperse particles in the flow are usually used as the initial parameters.

In calculating the force of interaction of spherical bubbles with the carrier flow of an ideal incompressible liquid (with allowance for the mutual influence of the bubbles), the binary distribution function, which shows the probability of relative positioning of two disperse particles in the liquid, was used in [3] as a parameter. By varying the form of this function, Kroshilin and Kroshilin [3] calculated the averaged force for both random and ordered distributions of particles.

In [1], the model parameters are the geometric parameters of the cell. For a spherical cell, these are the dimensionless parameter $\eta_{\rm c}$ characterizing the ratio of the cell size to the length of the zone of the carrier-phase flow disturbed by the disperse particle. Arbitrary definition of the initial parameter has no effect on the structure of relations determining, e.g., the parts of the fluctuating component of the surface-force tensor but introduce some uncertainty into the numerical value of the coefficient ηc. The choice of this coefficient takes into account the influence of the flow structure on the calculation result.

Hence, the complexity of the objects and processes under study not only makes their modeling difficult but also hinders obtaining and processing of experimental information necessary for correct selection of the initial parameters of the model. Indeed, it is possible but extremely difficult to experimentally determine such a model parameter as, e.g., the binary probability distribution function. Therefore, the parameters of numerical models are often found and optimized in an indirect manner, by comparing the properties of the model and numerical results with the data of experiments where the measured parameters are the mass flow rates, gas contents, pressure distributions, etc. (these quantities are not parameters of two-phase flow models).

Institute on Exploitation of Nuclear Power Stations, Moscow 109507. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 45, No. 1, pp. 39–45, January–February, 2004. Original article submitted January 30, 2003; revision submitted May 14, 2003.

The objective of the present work is to analyze the wave properties of the system of equations of a gas–liquid flow, which was obtained with allowance for fluctuating components of kinetic energy and stresses in the carrier (liquid) phase. The form of coefficients at fluctuating terms is chosen such that the wave properties of the model do not contradict the consequences of the Zuber–Findlay empirical law [4, 5]. In addition, the form of the system should not contradict the procedure of averaging of hydrodynamic "microequations" [1] from which it is derived. It is important that the bubbly flow model [6] possesses the properties that take into account the Zuber–Findlay law, though the possibility of deriving equations of motion with such properties were not discussed in [6]. Therefore, it is of interest to consider the procedure of obtaining the equations of motion at least in the approximation of phase incompressibility. Construction of a system with the properties mentioned above will allow combining the advantages of two-fluid models (possibility of taking into account the inertia of each phase and the effect of virtual mass, more detailed allowance for acoustic processes in the flow (see, e.g., [6]) as compared to the one-fluid model) and exact modeling of propagation of kinematic waves, which is successfully performed in one-fluid modeling.

The analysis is important because correct allowance for the Zuber–Findlay law is hindered in modeling of heterogeneous flows within the framework of a two-fluid system of equations of hydrodynamics of a heterogeneous flow (e.g., in TRAC or CATHARE computational codes), and this problem in the practice of numerical calculations is often solved within the framework of one-fluid modeling, where phase slipping U is specified directly from empirical data.

Basic Assumptions. The kinetic energy of small-scale motion (which is determined by the trace of the tensor of fluctuating stresses) and the fluctuating component of the surface stress tensor corresponding to the spatially one-dimensional averaged equation of motion of the liquid phase are presented as

$$K_1 = \alpha_2 \chi U^2/2; \tag{1}$$

$$\Pi_1 = \alpha_2 \psi U^2, \tag{2}$$

where α_2 is the volume gas content, $U = V_2 - V_1$ is the relative velocity of the phases, V_2 and V_1 are the macroscopic (averaged) velocities of the disperse and carrier phases, respectively, and χ and ψ are the functions of the volume gas content; α_2, χ, and ψ are considered as the sought parameters in the present work. Their choice effectively takes into account the influence of the flow structure, possible deviations of the bubble shape from a sphere, and the effect of the difference between the pressure averaged over the bubble surface and the mean pressure in the liquid. In deriving Eqs. (3) and (4), phase transitions are ignored, the bubbles are considered as solid spherical particles, the phase are assumed to be incompressible, and the heat transfer between the phases is also ignored. Thus, the internal energy of each phase is considered to be unchanged. The effect of viscosity is taken into account later, in Eqs. (5) and (6).

System of Equations. With allowance for the above said, we write the momentum- and energy-balance equations for a two-phase flow as a whole:

$$\rho_1 \alpha_1 \frac{d_1 V_1}{dt} + \rho_2 \alpha_2 \frac{d_2 V_2}{dt} + \frac{\partial (P + \rho_1 \Pi_1)}{\partial z} = F_{\rm ext}; \tag{3}$$

$$\rho_1 \alpha_1 \frac{d_1 (V_1^2/2 + K_1)}{dt} + \rho_2 \alpha_2 \frac{d_2 (V_2^2/2)}{dt} - \frac{\partial C_1}{\partial z} = F_{\rm ext} V. \tag{4}$$

Here $d_k/dt = \partial/\partial t + V_k\, \partial/\partial z$, $k = 1, 2$ [the subscripts 1 and 2 refer to the liquid (carrier) and disperse phases, respectively], $\alpha_1 + \alpha_2 = 1$, P is the pressure, $W = \alpha_1 V_1 + \alpha_2 V_2$, and $C_1 = -PW - V_2 \rho_1 \Pi_1$ is the work of surface forces [1, Chapter 3]. If phases 1 and 2 are incompressible, Eqs. (3) and (4) can be transformed to

$$\frac{\partial (\rho_1 \alpha_1 V_1)}{\partial t} + \frac{\partial (\rho_1 \alpha_1 V_1^2 + \alpha_1 P + \rho_1 \Pi_1)}{\partial z}$$

$$= -P \frac{\partial \alpha_2}{\partial z} - \chi \rho_1 \alpha_1 \alpha_2 \Big(\frac{d_1 V_1}{dt} - \frac{d_2 V_2}{dt} \Big) + \Phi_\alpha U^2 \frac{\partial \alpha_2}{\partial z} + \Phi_u U \frac{\partial U}{\partial z} + F_1; \tag{5}$$

$$\frac{\partial (\rho_2 \alpha_2 V_2)}{\partial t} + \frac{\partial (\rho_2 \alpha_2 V_2^2 + \alpha_2 P)}{\partial z}$$

$$= P \frac{\partial \alpha_2}{\partial z} + \chi \rho_1 \alpha_1 \alpha_2 \Big(\frac{d_1 V_1}{dt} - \frac{d_2 V_2}{dt} \Big) - \Phi_\alpha U^2 \frac{\partial \alpha_2}{\partial z} - \Phi_u U \frac{\partial U}{\partial z} + F_2, \tag{6}$$

where

$$\Phi_\alpha = \rho_1\Big\{\frac{d(\alpha_2\psi)}{d\alpha_2} - \alpha_2\psi - \frac{\alpha_1}{2}\Big[\chi(1-3\alpha_2) + \alpha_1\alpha_2\,\frac{d\chi}{d\alpha_2}\Big]\Big\}; \tag{7}$$

$$\Phi_u = \rho_1\Big\{2\alpha_2\psi + \alpha_1\alpha_2\Big[\psi - \frac{\alpha_1}{2}\Big(3\chi + \alpha_2\,\frac{d\chi}{d\alpha_2}\Big)\Big]\Big\}. \tag{8}$$

It is seen from (5), (6) that, if the fluctuating terms of system (1) are ignored (for $\psi = 0$ and $\chi = 0$), this system is the equations of momentum in the form of the Euler equations with allowance for the Rakhmatulin force. Allowance for kinetic energy of small-scale motion in expressions (3) and (4) ensures allowance of the force of virtual masses $\chi\rho_1\alpha_1\alpha_2(d_1V_1/dt - d_2V_2/dt)$ in (5) and (6), and the value $\chi = 1/2$ for $\alpha_2 = 0$ obtained in [1] corresponds to the correct limiting value of the coefficient of virtual masses.

The terms in the right sides of (5) and (6) proportional to $\partial\alpha_2/\partial z$ and $\partial U/\partial z$ correspond to interphase forces caused by small-scale motion and collective interactions of disperse particles with the carrier flow [3]. The terms F_1 and F_2 in the right side of Eqs. (5) and (6) express the external volume forces, but in what follows we assume that

$$F_1 = \rho_1\alpha_1 g_z - F_{12}, \qquad F_2 = \rho_2\alpha_2 g_z + F_{12}, \qquad F_{12} = -\alpha_1\alpha_2 K_\mu U\,|U|. \tag{9}$$

Here F_{12} is the force of interphase interaction due to viscosity and K_μ is the drag coefficient depending on the carrier flow regime around the disperse particles, volume gas content, disperse particle size, and carrier phase density. Relations (5) and (6) are the equations of motion for the phases of the bubbly flow.

Characteristic Properties of the System of Equations. Representing the velocity of each phase as a function of the total volume flow rate W and slipping U and eliminating terms containing $\partial P/\partial z$ from Eqs. (5) and (6), we obtain the following equation for phase slipping:

$$\frac{\partial U}{\partial t} + (W + UK_u)\,\frac{\partial U}{\partial z} - U^2 K_\alpha\,\frac{\partial\alpha_2}{\partial z} = F. \tag{10}$$

Here

$$K_u = \frac{\rho_1}{R}\Big(3(\psi - 0.5\alpha_1\chi) - 0.5\alpha_1\alpha_2\,\frac{d\chi}{d\alpha_2} + \frac{\alpha_1\rho_2}{\rho_1} - \alpha_2 + \chi(\alpha_1 - \alpha_2)\Big),$$

$$K_\alpha = \frac{\rho_1}{R}\Big(\frac{1}{\alpha_1\alpha_2}\,\frac{d[\alpha_1\alpha_2(0.5\alpha_1\chi - \psi)]}{d\alpha_2} + \frac{\rho}{\rho_1} + \chi\Big),$$

$$R = \rho_1\rho_2\,\frac{1 + \chi\rho_*/\rho_2}{\rho_*}, \quad \frac{1}{\rho_*} = \frac{\alpha_1}{\rho_1} + \frac{\alpha_2}{\rho_2}, \quad F = \frac{g^*(\rho_1 - \rho_2) - K_\mu U\,|U|}{R}, \quad g^* = \frac{dW}{dt} - g_z.$$

We consider Eq. (10) together with the equations of continuity for the liquid and gas phases:

$$\frac{\partial(\rho_1\alpha_1)}{\partial t} + \frac{\partial(\rho_1\alpha_1 V_1)}{\partial z} = 0; \tag{11}$$

$$\frac{\partial(\rho_2\alpha_2)}{\partial t} + \frac{\partial(\rho_2\alpha_2 V_2)}{\partial z} = 0. \tag{12}$$

If the phases are incompressible, Eqs. (11) and (12) yield the condition of independence of the volume flow rate W of the coordinate [which allows us to consider W as the boundary condition (a given function of time)] and the equation for the volume gas content

$$\frac{\partial\alpha_2}{\partial t} + \frac{\partial(\alpha_2 V_2)}{\partial z} = 0. \tag{13}$$

Taking into account that $V_2 = W + \alpha_1 U$, we obtain from (13)

$$\frac{\partial\alpha_2}{\partial t} + \alpha_1\alpha_2\,\frac{\partial U}{\partial z} + [W + (\alpha_1 - \alpha_2)U]\,\frac{\partial\alpha_2}{\partial z} = 0. \tag{14}$$

System (10) and (14) is closed. Its characteristic equation has two roots:

$$\lambda_1 = W + U[\alpha_1 - \alpha_2 + K_u + ((\alpha_1 - \alpha_2 - K_u)^2 - 4\alpha_1\alpha_2 K_\alpha)^{0.5}]/2; \tag{15}$$

$$\lambda_2 = W + U[\alpha_1 - \alpha_2 + K_u - ((\alpha_1 - \alpha_2 - K_u)^2 - 4\alpha_1\alpha_2 K_\alpha)^{0.5}]/2. \tag{16}$$

We require that the following obvious condition be satisfied as $\alpha_2 \to 0$:

$$\lambda_1 = \lambda_2 = V_2 \tag{17}$$

(V_2 is the velocity of the disperse phase). Indeed, as $\alpha_2 \to 0$, the velocity of propagation of gas-content perturbations in the flow should approach the velocity of gas bubbles. As follows from Eqs. (15) and (16) and from the definition of the functions K_u and K_α, condition (17) is satisfied if, as $\alpha_2 \to 0$, we have

$$\alpha_1\chi/2 - \psi \sim \alpha_2^m, \tag{18}$$

where $m > 1$. For $\alpha_2 = 0$, it follows from Eq. (18) that $\chi/2 = \psi$; $\psi = 0.25$ for $\chi = 1/2$. This value of ψ differs from that obtained in [1] for spherical bubbles and equal to 0.2, which can be explained by the influence of assumptions accepted in determining the quantities K_1 and Π_1.

The Zuber–Findlay relation [4]

$$V_2 = C_0 W + V_w \tag{19}$$

(C_0 and V_w are independent of W) yields the drift equation (see [5]). Indeed, we consider the particular case of Eq. (19) corresponding to $C_0 = 1$. Obviously, in this case, Eq. (19) yields the relation determining phase slipping as a function of the volume gas content:

$$U = U_0(\alpha_2). \tag{20}$$

The relation of the form (20) is rather generic. According to [7], where empirical data for calculating the relative phase velocity in an upward two-phase flow are given, the assumption $C_0 = 1$ is fairly justified for $\alpha_2 \leqslant 0.65$. Substituting relation (20) into (14), we obtain the expression known in the literature (see, e.g., [5, p. 295]) as the drift equation

$$\frac{\partial \alpha_2}{\partial t} + V_d \frac{\partial \alpha_2}{\partial z} = 0, \tag{21}$$

where $V_d = W + (\alpha_1 - \alpha_2)U_0 + \alpha_1\alpha_2\, dU_0/d\alpha_2$. Note, for $\alpha_2 \to 0$, the relation for V_d does not contradict condition (17).

As was indicated in [5, 8], the drift model correctly describes the wave processes in two-phase flows with incompressible phases. Moreover, an unsteady process such as the decay of an arbitrary discontinuity (without allowance for compressibility effects) was analyzed in [5] from the viewpoint of the drift model.

Under these circumstances, it seems reasonable to determine the parameters of Eqs. (10) and (14) [$\chi = \chi(\alpha_2)$ and $\psi = \psi(\alpha_2)$] based on the condition of agreement with the wave properties of the drift equation.

In the steady regime with a uniform distribution of the gas content and slipping of the phases, relation (10) is responsible for satisfaction of the equality $F = 0$, which, in the case of a proper choice of the coefficient K_μ (as was demonstrated in [9]), coincides with the empirical relation (20). The required agreement of the wave properties is observed under the condition

$$\lambda_1 = \lambda_2 = V_d \tag{22}$$

for $0 \leqslant \alpha_2 \leqslant 1$.

It follows from Eqs. (15) and (16) that Eq. (22) is valid if the functions χ and ψ satisfy the following system:

$$\begin{gathered} K_\alpha\Big(\chi, \psi, \frac{d\chi}{d\alpha_2}, \frac{d\psi}{d\alpha_2}\Big) = \alpha_1\alpha_2\Big(\frac{d\ln U_0}{d\alpha_2}\Big)^2, \\ K_u\Big(\chi, \psi, \frac{d\chi}{d\alpha_2}\Big) = \alpha_1 - \alpha_2 + 2\alpha_1\alpha_2\,\frac{d\ln U_0}{d\alpha_2}. \end{gathered} \tag{23}$$

For moderate values of α_2, for which the flow can be considered as bubbly, the solution of system (23) χ, ψ with the initial conditions $\chi(0) = 1/2$ and $\psi(0) = 1/4$ is monotonically decreasing positive functions. The dependence $\chi = \chi(\alpha_2)$ obtained by solving Eq. (23) numerically under the assumption that $U_0 = \text{const}/\alpha_1$ and $\rho_2 \ll \rho_1$ is plotted in Fig. 1 by curve 1. Curve 2 is the dependence

$$\chi = (1 - \eta c\alpha_2)/\alpha_1, \tag{24}$$

obtained in [1, p. 124] by the cell method. The best agreement of the solution of system (23) and dependence (24) is observed for the value of the cell-model parameter $\eta c = 3.5$. This value of ηc corresponds to fluctuating small-scale motion that covers a certain layer of the carrier phase near the disperse particle; the thickness of this layer is 82% of the cell size.

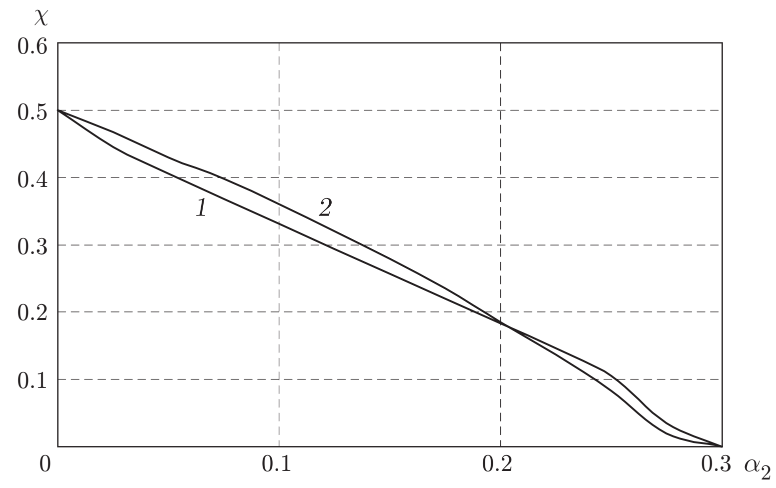

Fig. 1. Dependence $\chi(\alpha_2)$.

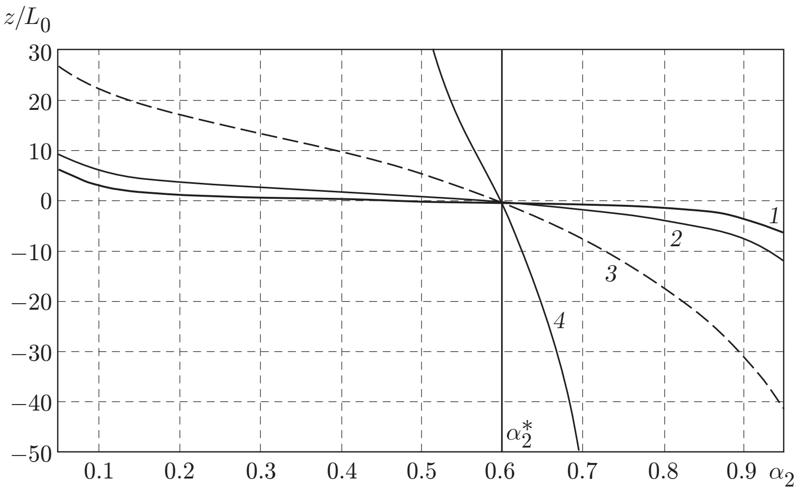

Fig. 2. Dependence $\alpha_2(z/L_0)$ for $t = 0$ (1), $3L_0/U$ (2), $20L_0/U$ (3), and $100L_0/U$ (4); L_0/U is the characteristic time of process stabilization in the drift model, determined in accordance with [6].

As an example of using this model, we consider the process of stabilization of a counterflow in a vertical channel. This example corresponds to the concepts about the final stage of an accident with the loss of coolant in the casing of the water-moderated power reactor, where water boiling with subsequent "hanging" of the water layer on a steam "cushion" occurs [9]. This "hanging" is broken because of its instability. Water moves downward, and steam moves upward. It was shown [6] that, if we choose Eq. (19) under the condition

$$C_0 = 1, \qquad V_w = C(1 + (\rho_2/\rho_1)^{0.5}\alpha_2/\alpha_1) \tag{25}$$

as the closing relation in modeling within the framework of the one-fluid model, we obtain a steady uniform distribution of the gas content with a value of α_2^* corresponding in the theory of kinematic waves [7] to the Wallis formula for flow choking:

$$W_1^{0.5} + (\rho_2/\rho_1)^{0.25}W_2^{0.5} = C^{0.5}.$$

Here W_1 and W_2 are the mean-volume flow rates of the liquid and gas, respectively, and C is the coefficient determined by test conditions. This means that the correct result can be obtained comparatively easily within the framework of the one-fluid model with a closing relation of the form (19). In two-fluid modeling, which seems to be more exact and detailed and where each phase has its "own" differential equation of motion, the correct choice of parameters, generally speaking, is not always possible even if the system of equations of the gas–liquid flow is hyperbolic. Within the present model, however, as was shown above, it is possible to set the required wave properties. Figure 2 shows the distributions of the parameter α_2 along the channel for different times, which were obtained using this model. One can see that counterflow stabilization occurs at $\alpha_2 = \alpha_2^*$.

Conclusions. The present analysis shows that the allowance for fluctuating components in momentum- and energy-balance equations allows obtaining a closed system of two equations of motion and two continuity equations, which possesses prescribed characteristic properties. The use of this technique allows one to take into account the Zuber–Findlay empirical law within the framework of the two-fluid calculation model. The form of the fluctuating components does not contradict the results obtained by the cell model.

It should be noted that, for the solutions of the drift equation (21) $\alpha_2 = \alpha_2(z,t)$, relation (20) is also a solution of Eq. (10) if condition (23) is satisfied and the volume flow rate W varies linearly with time. Thus, within the framework of model (10)–(12), (23), the solutions of the drift equation $U = U_0(\alpha_2)$ obtained from the condition $F = 0$ can be considered as the Riemann solutions.

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